BC-2880

BCA (Semester-IV) Examination–2015

Mathematics-III

Time : Three Hours

Maximum Marks : **75**

Note :- Attempt questions from all the sections.

SECTION – A

(Short Answer Type Questions)

Note : Attempt **any ten** questions. Each question carries three marks. 3×10=30

1. Express the following in a+ib form where a and b are real :

$$\frac{2-3i}{4-i}$$

2. Find modulus and principal arguments of :

$1-\cos\alpha + i\sin\alpha$

3. Discuss the convergence of the sequence {un}, where

$$un = \frac{n+1i}{n}$$

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4. Discuss the nature of the series :

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots\ldots\infty.$$

5. Prove that

$$\frac{d}{dt}\left(\overline{F}\times\overline{G}\right)=\overline{F}\times\frac{d\overline{G}}{dt}+\frac{d\overline{F}}{dt}\times\overline{G}$$

6. Calculate the directional deviative of the function $\phi\ (x,\ y,\ z)\ =\ xy^2+yz^3$ at the point $(1,\ -1,\ 1)$ in the direction of $(3,\ 1,\ -1)$.

7. Find Fourier expression for

$$f(x) = \pi - x \quad \text{for} \quad 0 < x < 2\pi$$

8. Solve the differential equation

$$x^4\frac{dy}{dx}+x^3y=-\sec(xy)$$

9. Solve the equation

$$\frac{dy}{dx}=\frac{y}{x}+x\frac{\sin y}{x}$$

10. Solve the following

$$(D^2+4)\,y = 3x\sin x$$

11. Solve

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^4$$

12. Test the series for convergence :

$$2/1^p + 3/2^p + 4/3^p + 5/4^p + \ldots.$$

13. Test the convergence of the series :

$$\frac{1}{2}x+x^2+\frac{9}{8}x^3+x^4+\frac{25}{32}x^4+\ldots\infty.$$

14. Find the normal vector of the curve at any point t :

$$x = 3\cos t,\quad y = 3\sin t,\quad z = 4t.$$

15. Prove that

$$\nabla\times(\overline{F}\times\overline{G})=\overline{F}(\Delta\cdot\overline{G})-\overline{G}(\nabla\cdot\overline{F})+(\overline{G}\cdot\Delta)\overline{F}-(F\cdot\Delta)\overline{G}$$

SECTION – B

(Long Answer Type Questions)

Note : Attempt **any three** questions. Each question carries 15 marks. 15×3=45

1. Write the definition of the following with examples :

(a) Monotonic Sequences.

(b) Convergent Series.

(c) Function of a complex variable.

2. Suppose $\vec{U}$, $\vec{V}$ and f are continuously differentiable fields. Then div $(\vec{U} \times \vec{V}) = \vec{V} \cdot \text{curl}\, \vec{U} - \vec{U} \cdot \text{curl}\, \vec{V}$.

3. Find the Fourier series for

$$f(x) = \begin{cases} -\pi & , -\pi < x < 0 \\ x & , 0 < x < \pi \end{cases}.$$

Deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$$

4. Find the Fourier half-range cosine series function :

$f(t) = 2t, \; 0 < t < 1$

$\;\;\;\;\;\; = 2(2-t), \; 1 < t < 2.$

5. Solve : $\dfrac{dy}{dx} = \dfrac{x + 2y - 3}{2x + y - 3}$.

6. Test the convergence of the series :

$$x + \frac{2^2 x^2}{\underline{|2}} + \frac{3^3 x^3}{\underline{|3}} + \frac{4^4 x^4}{\underline{|4}} + \ldots\ldots \infty.$$

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BCA IVth Semester Exam.-2016

Mathematics III

Time : Three Hours

Maximum Marks : **75**

Note : Attempt questions from all sections.

SECTION - A

(Short-answer Type Questions)

Note : Attempt **any 10** questions. Each question carries 3 marks. 3×10=30

1. Express $(2+3i)/(4+5i)$ in the form $x+iy$.

2. Prove $\lim_{n\to\infty}(1+2/n)^n = e^2$.

3. If $r=(\ell+1)i+(\ell^2+\ell+1)j+(\ell^3+\ell^2+\ell+1)k$

   Find $dr/d\ell$ and $d^2r/d\ell^2$.

4. Find the directional derivative of $f(x,y,z)=x^2yz+4xz^2$ at the point $(1,-2,-1)$ in the direction of the vector $2i-j-2k$.

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5. If $V = e^{xyz}(i + j + k)$, find Curl V.

6. Prove it "Every convergent sequence of real numbers is a cauchy sequence."

7. Solve $y - x\dfrac{dy}{dx} = a\left(y^2 + \dfrac{dy}{dx}\right)$.

8. Test the covergence of the series $\sum\left(1+\frac{1}{n}\right)^{-n^2}$.

9. Define the Periodic function.

10. Test the convergene of the series

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\ldots\ldots\ldots\ldots$$

11. Solve $x\cos x(dy/dx) + y(x\sin x + \cos x) = 1$.

12. Find the fourier series of the function $f(x) = x,\ -\Pi < x < \Pi$.

13. Solve $(x^2 - ay)dx - (ax - y^2)dy = 0$

14. If $f = f1i + f2j + f3K$ is differentiable vector point function, then

$$\text{Curl } f = \left(\frac{\partial f_3}{\partial y}-\frac{\partial f_2}{\partial z}\right)i+\left(\frac{\partial f_1}{\partial z}-\frac{\partial f_3}{\partial x}\right)j+\left(\frac{\partial f_2}{\partial x}-\frac{\partial f_1}{\partial y}\right)K.$$

15. Solve $(D^4 - K^4)y = 0$

SECTION - B

(Long answer type questions)

Note : Attempt **any three** questions. Each question carries 15 marks. (15x3=45)

1. Find the Fourier half-range cosine series of the function :

$$f(x) = \begin{cases} 2t, & 0 < t < 1 \\ 2(2-t), & 1 < t < 2 \end{cases}$$

2. Solve

$$(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (x+1)e^x.$$

3. Prove that

$$\nabla \times (\nabla \times A) = \nabla(\nabla.A) - \nabla^2 A$$

4. If <Sn> is a sequence of positive terms then $\lim_{n\to\infty} Sn^{1/n} = \lim_{n\to\infty}\dfrac{Sn+1}{Sn}$.
Provided the limit on the right hand side exists, whether finite as infinite.

5. If $a = \sin\theta i + \cos\theta j + \theta k$, $b = \cos\theta i - \sin\theta j - 3K$ and $C = 2i + 3j - 3k$, find $\dfrac{d}{d\theta}[a\times(b\times c)]$ at $\theta = \Pi/2$ and $d/d\theta\,(b\times c)$.

6. Obtain the fourier series of $f(x) = x + x^2,\ -\Pi < x < \Pi$ and Prove that

$$\frac{\Pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots$$

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BCA (Semester-IV) Examination–2017

Mathematics III

Time : Three Hours *Maximum Marks :* **75**

Note : Attempt questions from all sections.

SECTION - A

(Short-answer Type Questions)

Note : Attempt **any ten** questions. Each question carries 3 marks. 10x3=30

1. Express z = (1 - i) in modular amplitude form.

2. If $\left|\frac{z-1}{z+1}\right| = 2$. Prove that the locus of z on the argond plane is a circle.

3. Solve $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

4. (a) Find value of $\left|e^{i\theta}\right|$

   (b) Define least upper bound

   (c) Periodic function in short

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5. Prove that a sequence can not converges to more than one limit i - e limit of a sequence is unique

6. Show that the sequence <Sn> difined by

$Sn = \sqrt{n+1} - \sqrt{n}, if\ n \in N$ is convergent.

7. Show that <Sn> converges to e, where Sn is

$$Sn = \left(1 + \frac{1}{n+1}\right)^n$$

8. Every bounded monotonically increasing sequence converges, prove it.

9. Form the differential equation from

$y = Ae^{2x} + Be^{x} + C$ where A, B, C are constants.

10. Solve $(x + y)(dx - dy) = dx + dy$.

11. Find the divergence and curl of the vector field.

$V(x, y, z) = x^2 y^2 \hat{i} + 2xy \hat{j} + (y2 - xy) \hat{k}$

12. Test for the convergence of the series

$$\Sigma \frac{\sqrt{n}}{n^2 + 1}$$

13. Show that if r = asin wt + b.cos wt, where a, b, w are constant. Then $\frac{d^2r}{dt^2} = -\omega^2 r$ and

$r x \frac{dr}{dt} = -wa\ x\ b$

14. If $r = |r|$ where $r = x\hat{i} + y\hat{j} + 2\hat{k}$, then prove that

$\nabla \log |r| = \frac{r}{r2}$.

15. Solve $(1-x^2)\frac{dy}{dx} + 2xy = x\sqrt{1-x^2}$

SECTION - B

(Long Answer type questions)

Note : Attempt **any three** questions. Each question carries 15 marks. 15x3=45

1. Find the fourier series expansion for f (x) if

$$f(x) \begin{cases} -\pi\ ; & -\pi < x < \theta \\ x\ ; & 0 < x < \pi \end{cases}$$

deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \frac{?^2}{8}$$

2. State and prove Cauchy's first and second theorem on limits.

3. Solve $(D^2 - 3D + 2)y = 6e^{2x} + Sin 2x$.

4. (a) If <Sn> is a Cauchy's sequence, then <Sn> is bounded, prove it.

(b) Apply Cauchy's root test for the converge of series.

$$\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2 . x^2 + \left(\frac{4}{5}\right)^3 x^3 + \ldots\ldots\ldots\ldots$$

5. (a) Test the convergence of the series whose n^{th} term is $\frac{\sqrt{n+1} - \sqrt{n-1}}{n}$.

(b) A particle moves along the curve $x = 2t^2$, $y = t^2$, $z = 3t-5$, where t is time. Find the components of its velocity of acceleration at the time t = 1 in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$.

6. Find fourier series for f(x) in the internal $(-\lambda, \lambda)$, where

$$f(x) = \begin{cases} x + \lambda & ; 0 \le x \le \lambda \\ -x - \lambda & ; -\lambda \le x < 0 \end{cases}$$

and $f(x + 2\lambda) = f(x)$